National Defence Défense nationale

USE OF ENERGY BOUNDS TO ESTIMATE FIELDS AND COUPLING INSIDE A CAVITY WITH APERTURES

by

C.L. Gardner and A. Louie

DEFENCE RESEARCH ESTABLISHMENT OTTAWA

REPORT NO. 1261

Canada

March 1995
Ottawa

National Défense
Defence nationale

USE OF ENERGY BOUNDS TO ESTIMATE FIELDS AND COUPLING INSIDE A CAVITY WITH APERTURES

by

C.L. Gardner and A. Louie
Electronic Countermeasures Section
Electronic Warfare Division

DEFENCE RESEARCH ESTABLISHMENT OTTAWA
REPORT N0. 1261

PCN
041LT

March 1995
Ottawa

ABSTRACT

This report investigates the use of energy bounds to estimate: the amount of energy that can be transmitted through an aperture when exposed to an EM pulse; the fields inside a cavity with one or more apertures; and the coupling of energy to a wire inside a cavity with apertures. New bounds have been determined for the energy that can be coupled from a damped-sine wave, a rectangular RF pulse and an inverse-exponential pulse.

It is shown that the use of energy bounds is a useful approach to the estimation of interior fields and coupling to systems inside a structure with apertures. This approach has potential for simplifying the hardening of electronic systems.

RÉSUMÉ

Ce rapport examine l'utilisation des limites supérieures d'énergie pour déterminer: la quantité d'énergie tramsmise par des ouvertures lorsqu'elles sont exposés à des impulsions électromagnétiques; les champs électromagnétiques à l'intérieur d'une cavité avec des ouvertures; et la couplage à un cable à l'intérieur d'une cavité avec des ouvertures. Les limites supérieures d'énergie ont été dévelopées pour des impulsions sinusoïdale amortie, des signaux RF rectangulaire, et des signaux exponentiel.

On a démontré que l'utilisation des limites supérieure est une approche efficace pour determiner les champs intérieur et la couplage à des systèmes à l'intérieur d'une structure. Cet approche peut être efficace pour le durissement électromagnétique des systèmes électronique.

EXECUTIVE SUMMARY

The electromagnetic (EM) vulnerability of CF vehicles, weapons systems, and communications systems is of concern. Potential threats include natural sources, such as lightning and electrostatic discharge, as well as a variety of man-made sources that includes the "normal" operational EM environment produced by high power radars, EW transmitters, and communications systems. At the same time, technological advances such as the use of a composite materials for construction; and increased miniaturization of electronics are contributing to increased EM susceptibility of many systems.

Analysis of the effects of external fields on military electronic systems is an important part of the hardening process. For large systems, such as a ship or aircraft, extensive computing facilities are needed to calculate the interior fields inside even a simplified model. In this report, an alterative approach based on the use of energy bounds is examined. It is shown that this method has potential for simplifying the EM hardening of military systems.

TABLE OF CONTENTS

1. BACKGROUND

A knowledge of the interaction and coupling of electromagnetic fields with large, closed structures such as aircraft, ships, missiles and buildings is critical for a designer to ensure that installed electronic equipment is compatible with the external electromagnetic environment. The external interaction problem usually involves the calculation of the surface currents induced on a simplified wire or patch model of the structure and subsequent calculation of the interior fields [1]. When the structure become electrically large, the calculation of the induced surface currents and fields requires extensive computer resources even when the model is simplified to the maximum extent and maximum use of symmetry is used. For example, calculations of the penetration of fields into the bridge of the CPF [1] took about 18 hours on a VAX 6000 at each frequency and was limited to a maximum frequency of about 200 MHz.

Because of the requirement for extensive computing facilities to calculate the fields inside even a simplified model of a large structure, it is usually not possible to include interior equipment in the calculation. When this is not possible, the coupling of fields to electronic systems is normally carried out in two stages. The first step is to calculate the interior fields using a simplified model that neglects the interior components. The second step is then to calculate the coupling of these fields to the electronic systems of interest and to estimate system susceptibilities.

The above approach has the following difficulties:

- if there is strong coupling to the interior fields, the Q of the structure will be lowered and the fields calculated will be overestimated;

- the coupling calculated using this two-stage approach can violate energy conservation (ie. the calculated coupled energy is more than that which can be transmitted through the apertures in the structure).

- neglect of minor features often gives models a higher symmetry than what exists in reality in order to reduce computation time. Because these models are expected to have higher Q-factors, the interior fields may be overestimated.

Because of the difficulties outlined in the computational approach described above, an alternative approach based on the use of energy bounds has been proposed by Lee [2], Warne and Chen [3,4] and others [5]. In this approach, one attempts to determine the maximum energy/power that can couple to an electronic system inside a structure with apertures. This approach recognises that it is virtually impossible to model all of the detail of any real world system and attempts to find a solution (in terms of an upper bound) that is independent of these system details.

In the remainder of this report, the use of bounds is explored as a method of estimating interior fields and coupling to systems inside of a structure with apertures.

2. BOUNDS ON ENERGY TRANSMITTED THROUGH AN APERTURE

2.1 APERTURE THEORY

Consider the penetration of the electromagnetic field through an aperture in an infinite planar screen, assumed to be perfectly conducting and vanishingly thin , as shown in Figure 1. Previous work has shown that this problem is most conveniently treated using the equivalence principle [6]. Using this principle, the penetration of the fields through the aperture is determined by considering the screen to be continuous and by placing appropriate magnetic surface currents, $\vec{M}_s$, over the aperture (Figure 2). The magnetic field ($\vec{H}^-$) on the left (illuminated) side can be written [7]

$$\vec{H}^-(\vec{r})=\vec{H}^{sc}-\frac{i\omega}{k^2}[k^2\vec{F}(\vec{r})+\nabla(\nabla\cdot\vec{F}(\vec{r}))]. \tag{1}$$

In this expression, $\vec{F}(\vec{r})$ is the electric vector potential given by

$$\vec{F}(\vec{r})=\frac{\epsilon}{4\pi}\iint_A 2\vec{M}_s(\vec{r})G(\vec{r},\vec{r}')ds' \tag{2}$$

where

$$\vec{M}_s(\vec{r}) = \vec{u}_z \times \vec{E}_t , \tag{3}$$

and

$$G(\vec{r},\vec{r}') = \frac{e^{-ik|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} . \tag{4}$$

In these equations, $\vec{H}^{sc}$ is the short circuit magnetic field [7], k is the propagation constant and ω is the angular frequency.

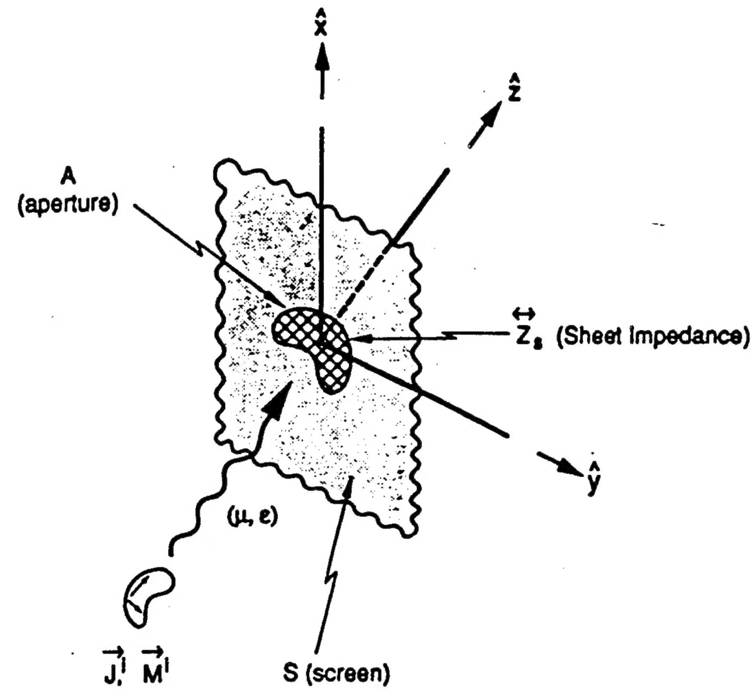


Figure 1 - The Penetration of Electromagnetic Fields Through an Aperture

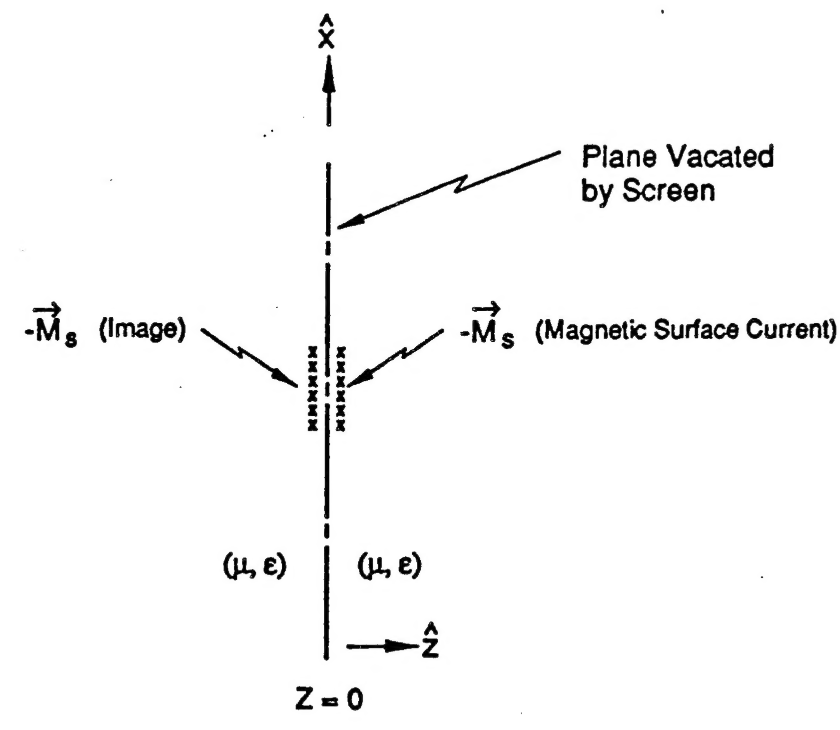


Figure 2 - The Equivalent Source Distribution for the Shadow Side of the Aperture

On the right hand side of the screen , the magnetic field, $\vec{H}^+$, is given by,

$$\vec{H}^+(\vec{r}) = \frac{i\omega}{k^2}[k^2\vec{F}(\vec{r}) + \nabla(\nabla\cdot\vec{F}(\vec{r}))]. \tag{5}$$

The electric fields in the two half spaces are given by the relationships,

$$\vec{E}^-(\vec{r}) = \vec{E}^{sc} - \frac{1}{\epsilon}\nabla\times\vec{F}(\vec{r}) \tag{6}$$

and

$$\vec{E}^+(\vec{r}) = \frac{1}{\epsilon}\nabla\times\vec{F}(\vec{r}) \tag{7}$$

where $\vec{E}^{sc}$ is the short circuit electric field [7].

When the aperture is open, the transverse components of the magnetic fields are continuous and the following relationship can be written [7]

$$\lim_{z\to 0}(\vec{H}^-(\vec{r})\times \vec{u}_z) = \lim_{z\to 0}(\vec{H}^+(\vec{r})\times \vec{u}_z) \tag{8}$$

which from equations (1) and (5) leads to the relationship

$$\vec{H}^i\times \hat{z} = \frac{i\omega}{k^2}[\nabla_t\nabla_t\cdot\vec{F}(\vec{r}) + k^2\vec{F}(\vec{r})] \times \vec{u}_z . \tag{9}$$

These equations show that, by using the incident magnetic field, $\vec{H}^i$, the equivalent magnetic currents, $\vec{M}_s$, can be found by solving (9) and the EM fields in both half spaces can be determined.

2.2 SMALL APERTURE THEORY

The electromagnetic field that penetrates through an electrically small aperture can be approximated [8] by the radiation from a magnetic dipole moment, $\vec{p}_m$, and an electric dipole moment, $\vec{p}_e$, both located at the centre of the aperture. These dipole moments are related [8] to the magnetic surface current, $\vec{M}_s$, by the expressions

$$\vec{p}_e = -\frac{\epsilon}{2}\iint_A \vec{r}'\times\vec{M}_s(\vec{r}')ds' \tag{10}$$

and

$$\vec{p}_m = -\frac{i}{\omega\mu}\iint_A \vec{M}_s(\vec{r}\,')ds' . \tag{11}$$

The above relationships show [9] that the magnetic and electric dipole moment are related to the irrotational component and the solenoidal component of $\vec{M}_s$ respectively.

The equivalent dipole moments, $\vec{p}_e$ and $\vec{p}_m$, are proportional to the incident field strengths E_z^i and $\vec{H}_t^i$ and electric and magnetic polarizabilities, α_e and $\ddot{\alpha}_m$, can be defined by

$$\vec{p}_e = \vec{u}_z\, \epsilon\alpha_e E_z^i \tag{12}$$

and

$$\vec{p}_m = \ddot{\alpha}_m \cdot \vec{H}_t^i . \tag{13}$$

In these expressions, E_z^i is the normal component of the incident electric field and $\vec{H}_t^i$ is the tangential component of the incident magnetic field.

For a small, circular aperture, the electric and magnetic polarizabilities, α_e and α_m, are scalar and are given [10] by

$$\alpha_e = \frac{4}{3}a^3 \qquad \alpha_m = -\frac{8}{3}a^3 \tag{14}$$

where a is the radius of the aperture. Analytical expressions have also been given [11] for small, elliptical apertures and numerical results have been given by De Meulenaere and van Bladel [12] for a variety of aperture shapes.

2.3 APERTURE CROSS-SECTIONS

The total energy absorbed and scattered by an aperture is related to the total aperture cross-section, σ_t, by the expression

$$W_{tot} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{|E(\omega)|^2}{\eta_0} \sigma_t(\omega)\, d\omega \tag{15}$$

where η_0 is the impedance of free space.

It has been shown by Born and Wolf [13] that the total cross-section, $\sigma_t(\omega)$, is related to the electric vector potential, $\vec{F}$, by

$$\sigma_t(\omega) = \frac{4\pi}{k^2} Re(\vec{u}_1 \cdot \vec{F}) \tag{16}$$

where Re denotes the real part of the quantity in the brackets and $\vec{u}_1$ is the unit vector in the direction of propagation of the incident field. In the far field, the scattered field, $\vec{E}_{sc}$, can be expressed in terms of $\vec{F}$ by the relationship

$$\vec{E}_{sc} = i\vec{F}\frac{e^{ikr}}{kr} . \tag{17}$$

If the scattering amplitude, $S(\omega)$, is defined as

$$S(\omega) = \frac{(\vec{u}_1 \cdot \vec{F})}{\omega^2} \tag{18}$$

then

$$\sigma_t(\omega) = 4\pi c^2 Re(S(\omega)) . \tag{19}$$

In terms of the scattering amplitude, the total energy, W_{tot}, can be written as

$$W_{tot} = \frac{2c^2}{\eta_0} \int_{-\infty}^{\infty} |E(\omega)|^2 Re(S(\omega)) d\omega . \tag{20}$$

Because the scattering process is causal, it follows [14] that $S(\omega)$ must be free of singularities in the $Im(\omega)>0$ half space. In addition, causality also implies that the real and imaginary parts form a Hilbert pair [15]. Use of this property gives [15]

$$Im(S(\omega)) = -\frac{2\omega}{\pi}\int_0^\infty \frac{Re(S(\zeta))}{\eta^2 - \omega^2}d\zeta = -\frac{\omega}{2\pi^2 c^2}\int_0^\infty \frac{\sigma(\zeta)}{\zeta^2 - \omega^2}d\zeta \,. \tag{21}$$

Taking the low frequency limit, $\omega \to 0$, leads [15] to

$$\int_0^\infty \sigma_t(\lambda)d\lambda = -4\pi^3 c^3 \omega^{-3} Im(\vec{u}_1 \cdot \vec{F}) \,. \tag{22}$$

The low frequency limit of (22) can be evaluated [3] in terms of the static electric and magnetic dipole moments of the aperture and it becomes

$$\int_0^\infty \sigma_t(\lambda)d\lambda = 2\pi^2\left(\alpha_{m_{22}} - \alpha_{e_{11}}\right) . \tag{23}$$

where $\alpha_{e_{11}}$ and $\alpha_{m_{22}}$ are principal components of the electric and magnetic polarizability tensors of the aperture.

General analytical expressions for the cross-section of even the simplest aperture shapes are not available. At low frequencies, the cross-section is of the general form $\sigma_t(\omega) \sim \omega^4$ and is also directly related to the electric and magnetic polarizabilities of the aperture. The following analytical expressions for the cross-section of a small, circular aperture have been given by Bethe [16] and Bouwkamp [17].

$$\sigma_t^{\perp} = \frac{64}{27\pi}a^6 k^4 \cos^2\theta^i \qquad \sigma_t^{\parallel} = \frac{64}{27\pi}a^6 k^4 \left[1 + \frac{1}{4}\sin^2\theta^i \right] . \tag{24}$$

where θ^i is the angle of incidence relative to a vector normal to the aperture.

At high frequencies, the aperture cross-section approaches the geometric area of the aperture and is independent of frequency and polarization, thus

$$\sigma_t \simeq A\cos\theta^i \, . \tag{25}$$

The shape of the cross-section when the wavelength of the incident wave is comparable to the size of the aperture is not well understood. Crevier and Auton [5] proposed the following general expression for the aperture cross-section

$$\sigma_t(\omega) = \frac{A_\infty \omega^4}{\left[(\omega-\omega_0)^2 + \sigma_0^2\right]\left[(\omega+\omega_0)^2 + \sigma_0^2\right]} \, . \tag{26}$$

These authors used this expression to derive a bound on the energy transmitted through an aperture with an incident field in the form of a "one minus an exponential" pulse.

The following, approximate, piece-wise linear expression has been given by Lee and Yang [18] for the cross-section of a narrow slot.

$$\begin{aligned} \sigma_t(f) &= l_s^2 (f/f_s)^4 \quad \text{for} \;\; f \le f_s = \frac{c}{2l_s} \\ &= l_s^2 (f_s/f)^2 \quad \text{for} \;\; f \ge f_s \, . \end{aligned} \tag{27}$$

The following section shows how other workers (especially Warne and Chen) have used the above methodology to define a bound on the total amount of energy that can be transmitted through an aperture.

2.4 BOUND ON ENERGY TRANSMITTED THROUGH AN APERTURE

2.4.1 A Step-Function Bound on EMP Coupling [3]

The step function can be considered as the limiting case of a double exponential pulse where the rise time approaches zero ($\beta \rightarrow \infty$) and the fall time approaches infinity ($\alpha \rightarrow 0$). In this case the spectrum of the incident field, $\vec{E}(\omega)$, can be written

$$\vec{E}(\omega) = \frac{i}{\omega} E_0 \vec{u}_1 . \tag{28}$$

Substitution of the above expression for $\vec{E}(\omega)$ into equation (15) yields the following expression

$$W_{tot} = \frac{1}{2\pi^2}\epsilon_0 E_0^2 \int_0^\infty \sigma_t d\lambda \tag{29}$$

From (23) the following upper bound is obtained

$$W_{tot} \leq \epsilon_0 E_0^2(\alpha_{m_{22}} - \alpha_{e_{11}}). \tag{30}$$

2.4.2 A Bound on EMP Coupling for a Double Exponential Pulse [4]

It is clear that use of the bound based on a step function will overestimate the energy transmitted through an aperture for a double exponential pulse with a finite rise and fall time. In a recent report, Crevier and Auton [5] investigated the effect of finite rise time on the energy that is radiated through an aperture. To do this they assumed the model for the aperture scattering amplitude presented in equation (26), in conjunction with a "one-minus-an-exponential" waveform, to determine a bound on the energy radiated through an aperture. The form of the scattering amplitude chosen by Crevier and Auton does not satisfy the causality requirement that it be free of singularities in the $Im(\omega)>0$ half space however, in spite of this apparent deficiency, the results obtained by Crevier and Auton are consistent with the results obtained by Warne and Chen described below.

In a recent paper, Warne and Chen [4] have developed a bound on EMP coupling for a double exponential pulse using a more rigorous treatment based on the methodology described in section 2.3. A summary of this method is given below.

The following double exponential form (Bell curve) is often used as an approximation to the EMP waveform because of its simplicity in spite of the fact that the first derivative is discontinuous.

$$E(t) = E_0(e^{-\alpha t} - e^{-\beta t})u(t) \tag{31}$$

The spectrum of this pulse is given by

$$E(\omega) = \int_{-\infty}^{\infty} E(t) e^{i\omega t} dt = \frac{E_0(\beta - \alpha)}{(\alpha - i\omega)(\beta - i\omega)} \tag{32}$$

The total energy scattered by and transmitted through the aperture is then given by

$$W_{tot} = \frac{2}{\eta_0} E_0^2 c^2 (\beta - \alpha)^2 Re \left[\int_{-\infty}^{\infty} \frac{S(\omega) d\omega}{(\alpha^2 + \omega^2)(\beta^2 + \omega^2)} \right] \tag{33}$$

Warne and Chen [4] have shown that, by using contour integration, the above expression reduces to

$$W_{tot} = 2\pi c^2 \frac{E_0^2}{\eta_0} \frac{(\beta - \alpha)}{(\beta + \alpha)} Re \left[\frac{S(i\alpha)}{\alpha} - \frac{S(i\beta)}{\beta} \right]. \tag{34}$$

2.4.3 A Bound on Coupling for a Damped-Sine Pulse:

Another waveform of practical interest, and one that has not been treated previously, is the damped sine wave described by the following equation

$$E(t) = E_0 \sin\omega_0 t \; e^{-at} \; u(t) \tag{35}$$

The spectrum of this pulse is given by

$$E(\omega) = \int_{0}^{\infty} E(t) \; e^{i\omega t} \; dt = \frac{E_0 \omega_0}{(-a + i\omega)^2 + \omega_0^2} \tag{36}$$

The total energy scattered by and transmitted through the aperture is then given by

$$W_{tot} = \frac{2E_0^2 c^2 \omega_0^2}{\eta_0} Re \left[\int_{-\infty}^{\infty} \frac{S(\omega)}{(a^2 - \omega^2 + \omega_0^2)^2 + 4a^2\omega^2} d\omega \right] \tag{37}$$

The integral in (37) can be evaluated using contour integration (Appendix A) yielding the following final result

$$W_{tot} = \frac{\pi E_0^2 c^2 \omega_0^2}{\eta_0 a^2} Re \left[\frac{S(\omega_0 + ia)}{a} - iS'(\omega_0 + ia) \right] . \tag{38}$$

2.4.4 A Bound on Coupling for a Rectangular RF Pulse:

The penetration of rectangular RF pulse through an aperture and subsequent coupling is of considerable interest because of the potential for threat from high power microwaves (HPM) and EM interference from on-board or nearby radars. A bound on the coupling for a rectangular RF pulse has not been presented previously in the literature.

A rectangular RF pulse can be represented in the time domain by

$$\begin{aligned} E(t) &= E_0 \cos\omega_0 t \qquad |t| \le a \\ &= 0 \qquad\qquad\quad\; otherwise \end{aligned} \tag{39}$$

The spectrum of this waveform is given by

$$E(\omega) = \int_{-\infty}^{\infty} E(t)\, e^{i\omega t}\, dt = -iE_0 \left[\frac{\sin a(\omega - \omega_0)}{\omega - \omega_0} - \frac{\sin a(\omega + \omega_0)}{\omega + \omega_0} \right] \tag{40}$$

Substitution of this expression into (15) gives

$$\begin{aligned} W_{tot} = \frac{2E_0^2 c^2}{\eta_0} Re \Bigg[&\int_{-\infty}^{\infty} \frac{\sin^2 a(\omega - \omega_0)}{(\omega - \omega_0)^2} S(\omega)\, d\omega \\ &+ \int_{-\infty}^{\infty} \frac{\sin^2 a(\omega + \omega_0)}{(\omega + \omega_0)^2} S(\omega)\, d\omega \\ &+ \int_{-\infty}^{\infty} \frac{2 \sin a(\omega - \omega_0) \sin a(\omega + \omega_0)}{\omega^2 - \omega_0^2} S(\omega)\, d\omega \Bigg] \end{aligned} \tag{41}$$

With some difficulty (Appendix B), the integrals in the above expression can be evaluated using contour integration. This leads to the following expression for W_{tot}.

$$W_{tot} = \frac{2\pi c^2 E_0^2}{\eta_0} \, Re \left[aS(\omega_0) + \left[a + \frac{\sin 2a\omega_0}{\omega_0} \right] S(-\omega_0) \right] \tag{42}$$

Noting that $Re[S(\omega)]$ is an even function allows W_{tot} to be rewritten as

$$W_{tot} = \frac{2\pi c^2 E_0^2}{\eta_0} \, Re \left[\left(2a + \frac{\sin 2a\omega_0}{\omega_0} \right) S(\omega_0) \right] \tag{43}$$

Substitution of equation (19) for σ_t into (40) then yields

$$W_{tot} = \frac{\sigma_t(\omega_0) E_0^2}{2\eta_0} \left[2a + \frac{\sin 2a\omega_0}{\omega_0} \right] = W_0 \left[1 + \frac{\sin 2a\omega_0}{2a\omega_0} \right]. \tag{44}$$

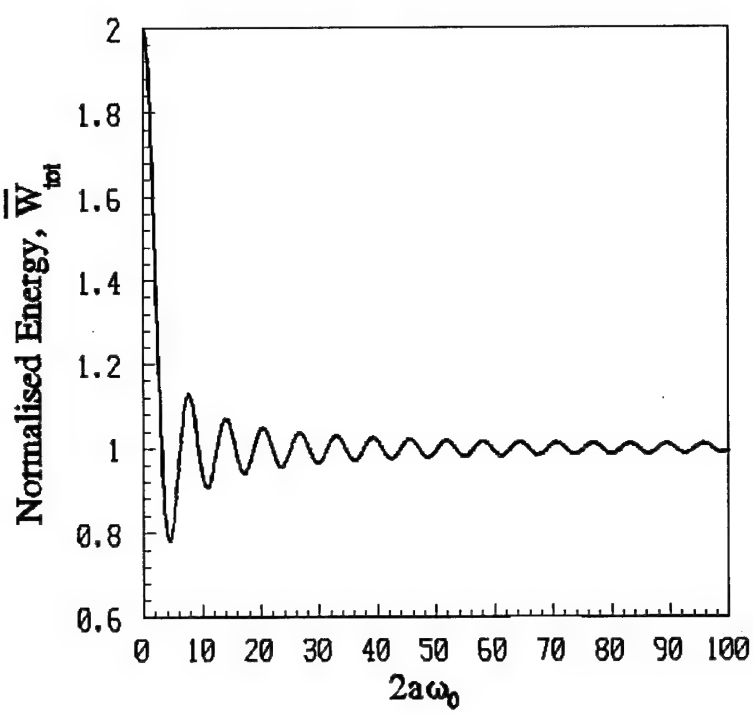


Figure 3 - Influence of Pulse Width on the Normalized Energy

Equation (44) has a clear physical interpretation. The term outside the first pair of brackets in (44) is the average power that would be transmitted through the aperture by a CW signal at frequency ω_0 and having amplitude E_0. The first term within the brackets, $2a$, is the pulse length and, when multiplied by the average power, gives a first order approximation to the total energy transmitted through the aperture. The second term represents a correction to the first term to account for the distribution of frequencies in a finite duration pulse. The influence of pulse width on the normalised energy, $\bar{W}_{tot} = W_{tot}/W_0$, is shown in Figure 3. If the pulse is long, the correction becomes negligible.

2.4.5 A Bound on Coupling for an Inverse Exponential (New NATO) Pulse

While the double exponential pulse(described above) is often used to describe a nuclear EMP because of its simplicity, it is discontinuous at time t=0 and has a singularity in its first derivative at this time. This discontinuity results in an overestimation of the high frequency content in the pulse. To overcome this difficulty, the following inverse double exponential wave-form has been proposed [19].

$$E(t) = E_0 \frac{d e^{\alpha t}}{1 + e^{\beta(t-t_p)}} . \tag{45}$$

The parameters needed to match the Bell Laboratory double exponential pulse are:

$\alpha = 10.30 \times 10^8\ \mathrm{s}^{-1}$
$\beta = 10.34 \times 10^8\ \mathrm{s}^{-1}$
$d = 1.16 \times 10^{-9}$
$t_p = 20$ ns
$E_0 = 50$ kV/m

The spectrum of this pulse is given [4] by

$$E(\omega) = E_0 d \frac{\pi}{\beta} \frac{e^{(\alpha + i\omega)t_p}}{\sin\left[(\alpha + i\omega)\frac{\pi}{\beta}\right]} \tag{46}$$

The total energy scattered by and transmitted through the aperture is then

$$W_{tot} = \frac{4c^2E_0^2\pi^2(\beta-\alpha)^{\frac{2\alpha}{\beta}-2}\alpha^{\frac{-2\alpha}{\beta}}}{\eta_0}\int_{-\infty}^{\infty}\frac{Re(S(\omega))}{\cosh\left(\frac{2\pi\omega}{\beta}\right)-\cos\left(\frac{2\pi\alpha}{\beta}\right)}d\omega\ . \tag{47}$$

The integral in (47) has been evaluated (Appendix C) by expanding the scattering amplitude, $S(\omega)$, as a Maclaurin series in ω. The integrals corresponding to the first three terms in this expansion were evaluated using contour integration. At low frequencies, it is known [16] that the cross-section varies as ω^4 indicating that the third term in the Maclaurin expansion is the first that should be retained. Substitution of an expression of this form into (47) gives results that are equivalent to those obtained by Warne and Chen [4]. At high frequencies, on the other hand, the cross-section approaches the geometric area of the aperture and it is independent of frequency. In this case the first term in the expansion will be dominant and the following approximate result is obtained for W_{tot}.

$$W_{tot} \simeq \frac{4c^2E_0^2\pi^2(\beta-\alpha)^{\frac{2\alpha}{\beta}-2}\alpha^{\frac{-2\alpha}{\beta}}(2\alpha-\beta)}{\eta_0\sin\left[\left(\frac{2\alpha}{\beta}-1\right)\pi\right]}Re(S(0))\ . \tag{48}$$

3. CAVITY BACKED APERTURES

3.1 INTRODUCTION

In a great many cases of practical interest, sensitive electronics are located within a metal structure (cavity) that contains one or more apertures. Examples include the avionics inside the fuselage of an aircraft, electronic equipment in the bridge of a ship or a circuit board inside a metallic case. Often we are required to know if external fields incident on these cavities will interfere with proper operation of the enclosed electronic systems.

In this section of the report, we examine the use of a power conservation approach to estimate the interior fields and to place a bound on the coupling of these fields to the electronics inside. This discussion largely follows the recent work of Hill et al [20] and Lee and Yang [18].

3.2 POWER CONSERVATION AND STEADY STATE CONDITIONS

Consider a plane wave incident on a structure (cavity) with apertures as shown in Figure 4. Initially we assume that the cavity is lossless except for energy reradiated back through the aperture. This implies that the cavity walls are perfectly conducting and that there is no other absorption of energy within the cavity. In this case, steady state fields are achieved when the power, P_t, being transmitted into the cavity is balanced by the power, P_r, being reradiated out of the cavity, that is when

$$P_t = P_r \, . \tag{49}$$

If the total transmission cross-section of the aperture is σ_t, then the power, P_t, transmitted into the cavity is

$$P_t = \sigma_t S_i \tag{50}$$

where S_i is the power density of the incident wave. If there are N apertures in the wall of the cavity, σ_t can be written as the sum

$$\sigma_t = \sum_{i=1}^{N} \sigma_{ti} \, . \tag{51}$$

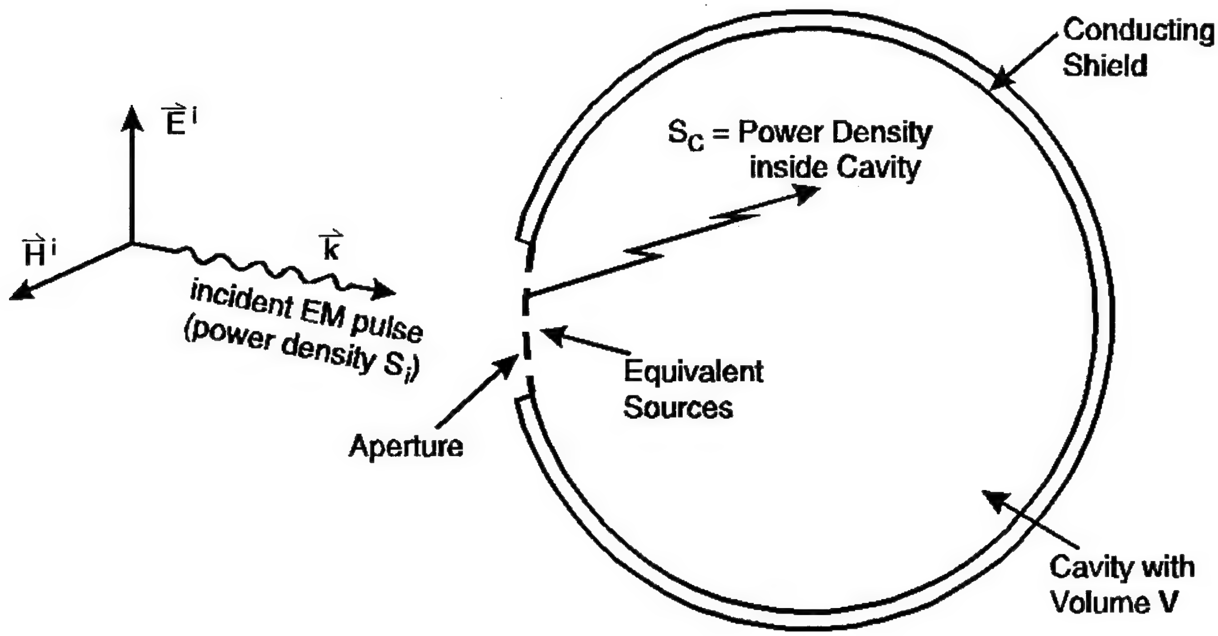


Figure 4 - Excitation of a Cavity with an Aperture

A relationship similar to (50) can be written for the power reradiated from the cavity. In many cases, it is necessary to express the fields inside the cavity as a superposition of plane waves and the cross-section that needs to be used is $\bar{\sigma}_r$, an average over this superposition. In general, the power leaking out of the cavity is given by

$$P_r = \bar{\sigma}_r \, S_c^{ap} \tag{52}$$

where S_c^{ap} is the power density of the field incident on the interior side of the aperture.

In certain limiting situations, the above power conservation relationship can be used to determine the fields in the interior of the cavity and the Q-factor of the cavity. Some special situations are examined below.

3.2.1 High Frequency Approximation

At high frequency, the field distribution in the cavity can be considered approximately uniform and the steady-state energy, U_s, can be written as

$$U_s = W\,V \tag{53}$$

where W is the average energy density and V is the cavity volume. The average field inside the cavity is related to the energy density by

$$W = \epsilon_0\, E^2\ . \tag{54}$$

In (54), ϵ_0 is the permittivity of free space and the electric and magnetic energies are assumed equal. The average power density in the cavity, S_c, is given by

$$S_c = \frac{E^2}{\eta_0} = cW\ . \tag{55}$$

At high frequencies, the superposition can be considered as consisting of plane waves of all incident angles and polarizations, but only plane waves that propagate toward the aperture contribute [20] to the leakage. The leakage power is

$$P_r = \langle\sigma_r\rangle\frac{S_c}{2} \tag{56}$$

where $\langle\sigma_r\rangle$ is the average over all incident angles and polarizations and S_c is the average power density inside the cavity.

Substitution of equations (50) and (56) into (49) yields the following relationship between S_c, the power density inside the cavity, and S_i, the power density of the incident wave.

$$S_c = \frac{2\sigma_t}{\langle\sigma_r\rangle}\, S_i\ . \tag{57}$$

The shielding effectiveness of the cavity can be defined in terms of the ratio of the incident and cavity power densities by the following relationship

$$SE = 10 \log\left(\frac{S_i}{S_c}\right) = 10 \log\left(\frac{\langle\sigma_r\rangle}{2\sigma_t}\right) . \tag{58}$$

This shows that the shielding effectiveness of the cavity is determined solely by the ratio of the two aperture cross-sections. In many cases σ_t is larger than $\langle\sigma_r\rangle$ and hence the shielding effectiveness is negative indicating that the interior fields are greater than the incident fields.

The Q of the cavity can be determined from the above relationships. Noting that the Q of the cavity is defined as

$$Q = \frac{2\pi \times \textit{steady state energy in cavity}}{\textit{energy loss per cycle}} = \frac{\omega U_s}{P_r} \tag{59}$$

where ω is the steady state frequency and U_s is the steady state energy, then this relationship, together with the steady state power condition, can be used to derive the following expression for cavity Q

$$Q = \frac{4\pi V}{\lambda\langle\sigma_r\rangle} . \tag{60}$$

This shows that the average transmission cross-section, $\langle\sigma_r\rangle$, for reradiation determines the cavity Q and that the cavity Q increases as the aperture size decreases. The above discussion assumes, however, that the cavity is lossless. In a practical situation, wall losses and absorption by systems inside the body will reduce cavity Q and need to be taken into account if interior fields are going to be determined accurately. The lossless situation described in this section corresponds closely however to numerical models that are normally used to calculate interior fields.

A case of interest is that of uniform random excitation as discussed by Hill et al [20]. In this case the averaged transmission cross-section is equal to the averaged cross-section for reradiation and the shielding effectiveness is 0 dB.

Expressions for the aperture cross-sections have been given in the literature for the cases where the aperture is electrically large or electrically small. In the following discussion, estimates of the shielding effectiveness are given for these two situations.

If the aperture is electrically large, the aperture cross-section can be written, from a geometrical optics approximation, as

$$\sigma_t = A \cos \theta^i \tag{61}$$

where A is the aperture area and θ^i is the angle of incidence of the plane wave with respect to a vector normal to the aperture.

The average cross-section for reradiation can be written as

$$\langle \sigma_r \rangle = \frac{1}{2\pi} \int_0^{2\pi} d\phi^i \int_0^{\frac{\pi}{2}} A \cos\theta^i \sin\theta^i \, d\theta^i = \frac{A}{2} \tag{62}$$

Substitution of these expressions into (58) yields the following for the shielding effectiveness of the cavity

$$SE = 10 \log\left(\frac{1}{4 \cos\theta^i}\right) . \tag{63}$$

This shows that at normal incidence ($\theta^i = 0$) the shielding effectiveness of the cavity is -6dB whereas at an angle of 75° the shielding effectiveness is 0 dB.

The transmission cross-sections for an electrically small circular aperture of radius a were given in section 2.3. Following Hill et al [20], the averaged reradiation cross-section, $\langle \sigma_r \rangle$, can be written as

$$\langle \sigma_r \rangle = \frac{1}{2} \int_0^{\pi/2} (\sigma_t^{\parallel} + \sigma_t^{\perp}) \sin\theta^i \, d\theta^i = \frac{16}{9\pi} k^4 a^6 . \tag{64}$$

For normal incidence, the shielding effectiveness becomes

$$SE = 10 \log\left(\frac{3}{4}\right) = -1.2 dB . \tag{65}$$

The above relationships indicate that, at high frequencies (ie. frequencies well above the lowest resonance frequency of the cavity when the field distribution inside the cavity can be considered uniform) the interior fields are expected to be of the same order as the incident field — at most a factor of two above incident. It is also of interest to note that, at this level of approximation, the fields are independent of aperture size and hence of cavity Q. This conclusion is supported by numerical calculations and measurements made, at DREO, of various structures including the CPF bridge [1] and the electronics box of missile [24] (Figure 5).

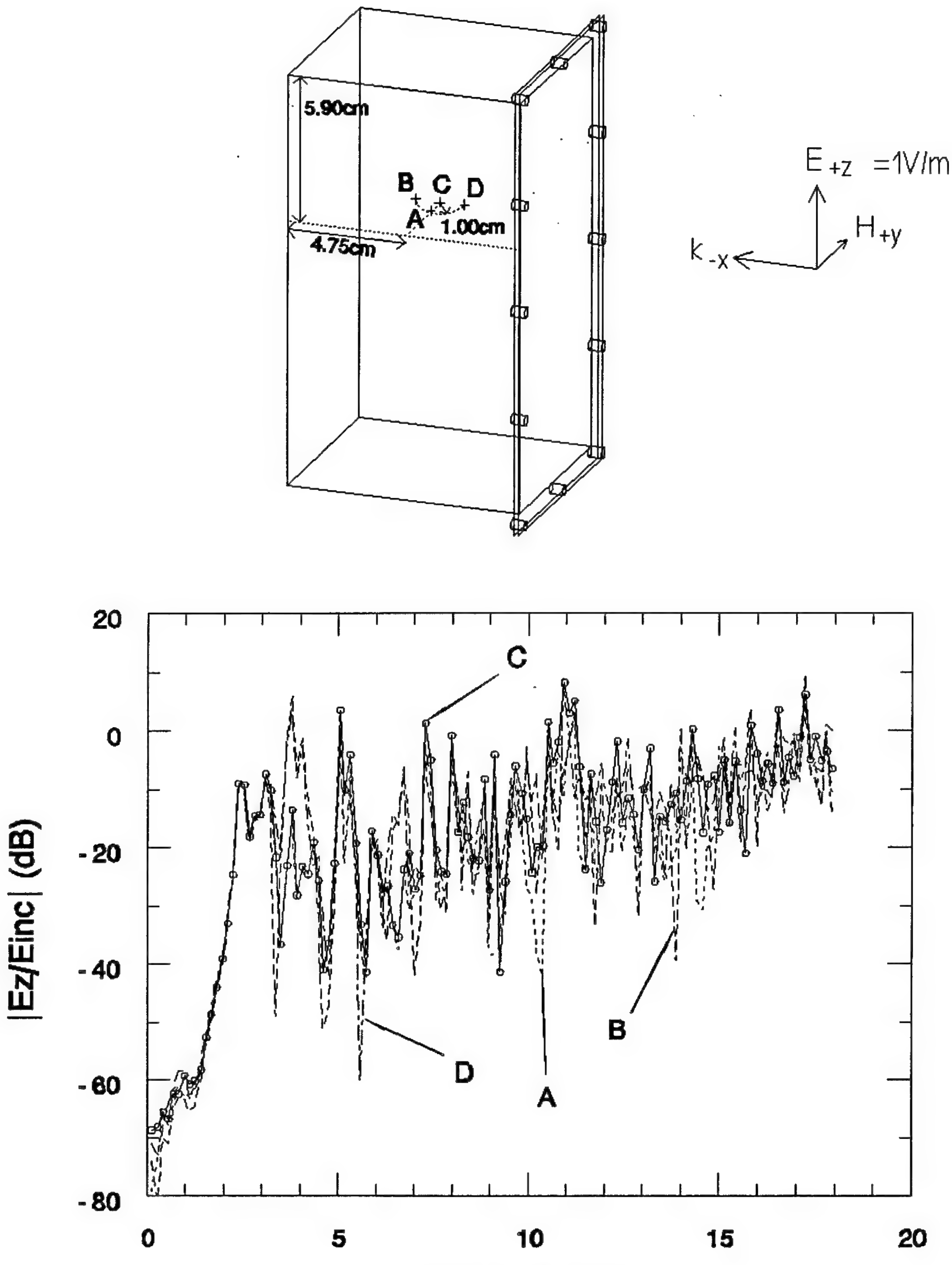


Fig 5 - Penetration of EM Fields into a Box with a Gasketed Lid [24].

3.2.2 Interior Fields at Cavity Resonance

For some simple geometries, the field structure inside the cavity at resonance can be expressed analytically. For example, the eigenvectors for the lowest TM mode of a circular cylindrical cavity are given [10] by

$$\vec{e}_m = J_0\left(2.405\frac{r}{b}\right)\vec{u}_z \qquad \vec{h}_m = J_1\left(2.405\frac{r}{b}\right)\vec{u}_\phi \tag{66}$$

where J_0 and J_1 are Bessel functions of the first kind, r is the distance from the centre of the cavity and b is the radius of the cavity. If we consider a plane wave normally incident on the endplate of a circular cylindrical cavity that contains a small, circular aperture as shown in Figure 6, then, by equating the transmitted and leakage powers, the following relationship is obtained for the shielding effectiveness of the cavity

$$SE = 10\log\left(\frac{4}{|J_0(2.405\frac{d}{b})|^2 + 4|J_1(2.405\frac{d}{b})|^2}\right) \tag{67}$$

where d is the distance of the centre of the aperture from the centre of the cavity. The effect of position of the aperture on the shielding effectiveness is illustrated in Figure 7.

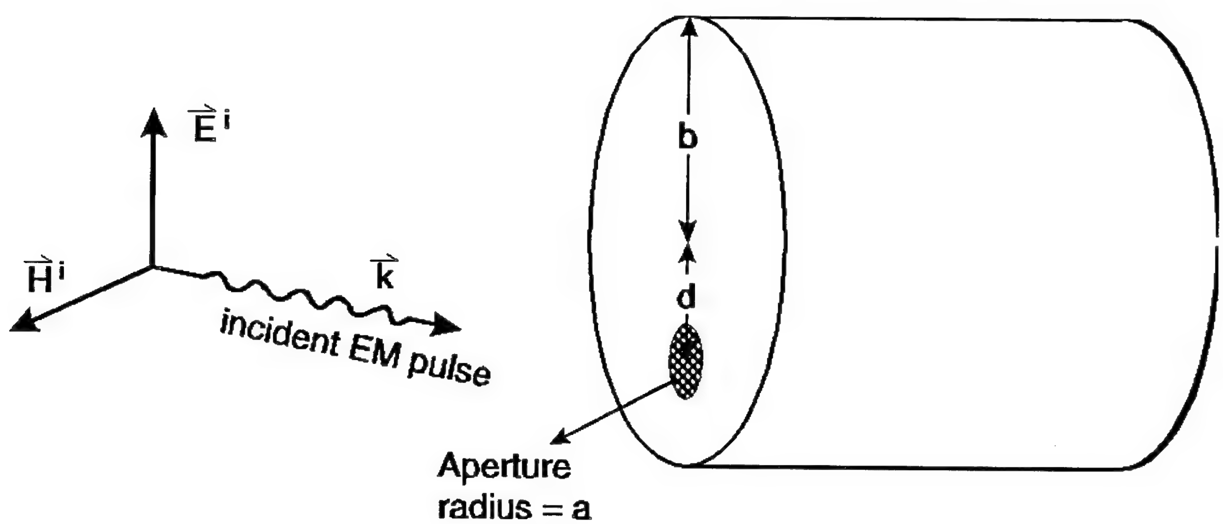


Figure 6 - Penetration into a Circular Cylindrical Cavity with Aperture

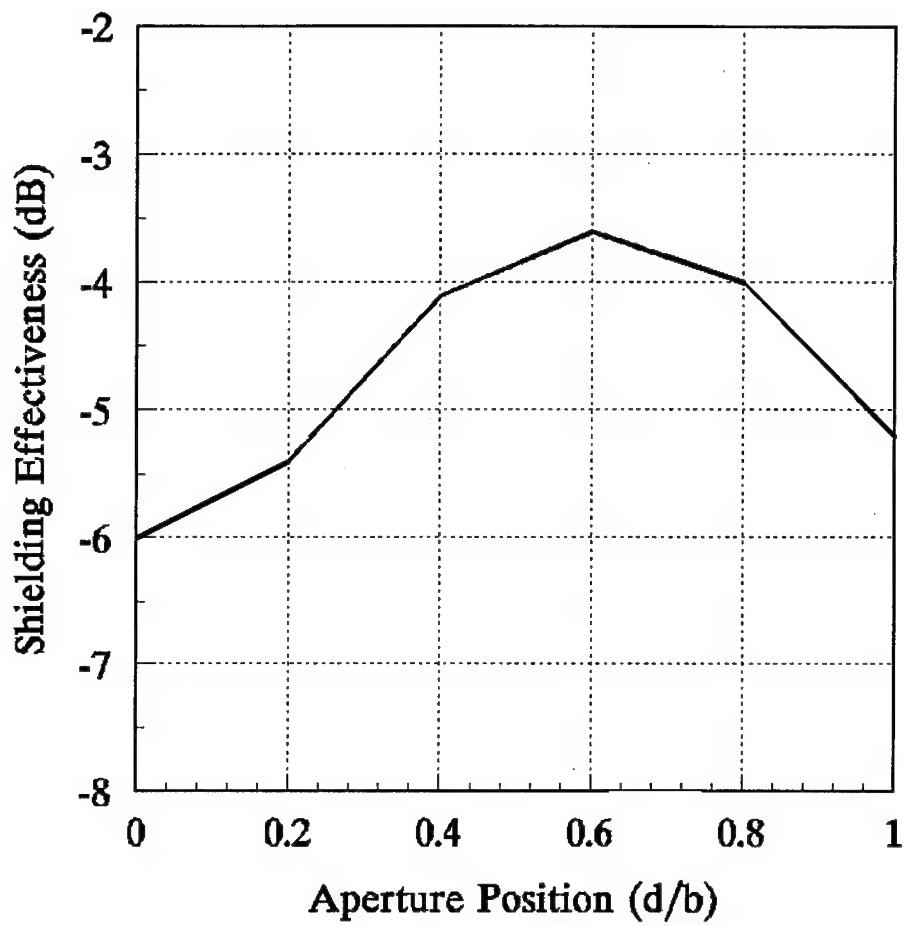


Figure 7 - Effect of Shielding Effectiveness on Aperture Position

3.2.3 Interior Fields at Low Frequencies.

As discussed by Stratton [21], the dominant fields in the vicinity of an aperture at low frequency are the inductive fields that decay as r^{-2} and r^{-3}. These fields account for the stored energy. There is a flow of energy into the cavity associated with these terms, however this flow averages to zero over a complete cycle. As a first order approximation to the fields inside the cavity at low frequency, the fields from the static magnetic and electric dipole moments of the aperture can be used. This assumes that the walls are far enough away that they do not influence the results. Alternatively, the more sophisticated mode matching approaches of Senior and Desjardins [22] or Mendez [23] can be used.

3.3 COUPLING TO WIRES INSIDE A CAVITY

Consider the situation shown in Figure 8 where a plane wave is incident on a cavity with an aperture in its surface. Inside the cavity is a wire that is connected to an electronic component such as an integrated circuit or transistor. An estimate (bound) on the energy that can couple to the wire is of interest to determine if upset or damage is likely to occur.

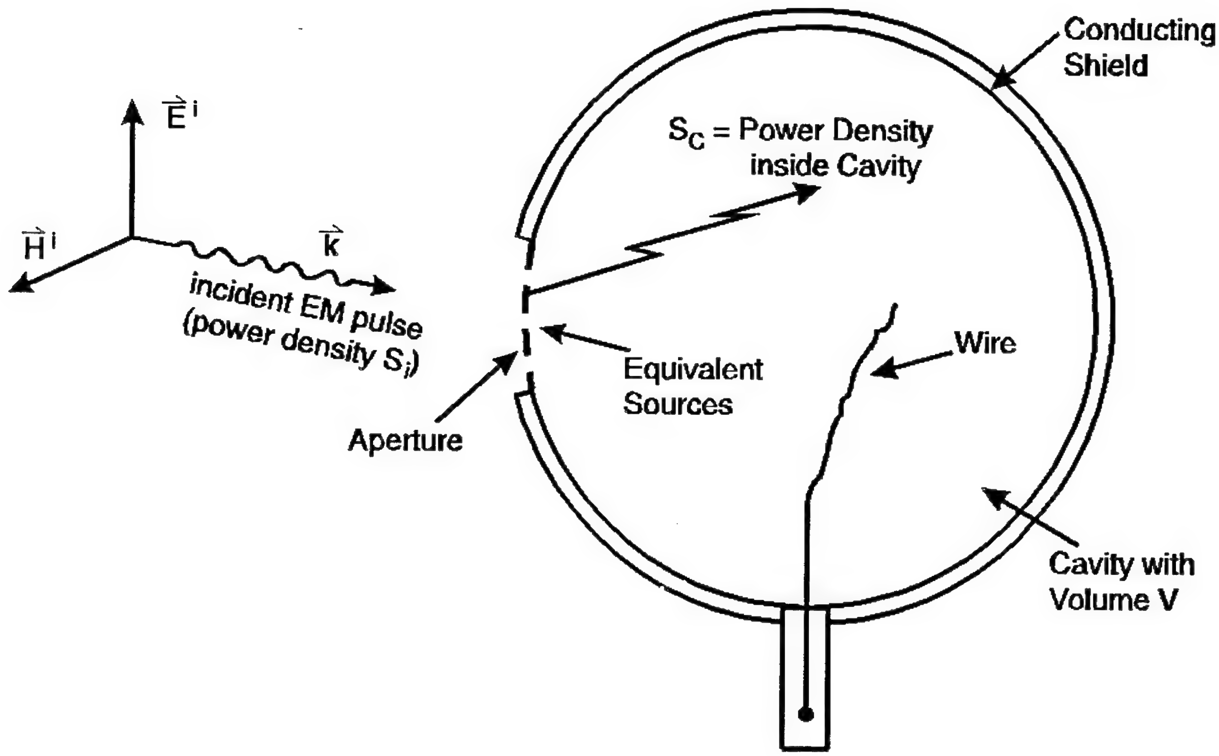


Figure 8 - Coupling to a Wire Inside a Cavity with an Aperture

Lee and Yang [18] have extended the power conservation approach outlined in section 3.2 to address this situation. Assuming that the field distribution inside the cavity is uniform (ie. high frequency approximation), then the power that can be coupled to the wire is given by

$$P_w = \sigma_w S_c = \sigma_p S_i \ . \qquad \textbf{(68)}$$

In this expression σ_w is the absorption cross-section that relates the power coupled to the wire to the interior field strength whereas σ_p is the cross-section that relates the power coupled to the wire to the incident field strength. σ_p includes the effects of the aperture.

The Q of the cavity will be modified by the absorption by the wire and can be written more generally as

$$Q = \frac{\omega W_T}{P_T} \tag{69}$$

where W_T and P_T are the total energy and total power dissipated in the cavity (including the loss in the wire, the power reradiated, and other losses such as wall losses). For conservation of power , the following condition applies

$$\sigma_T S_c = \sigma_t S_i \ . \tag{70}$$

Using the above relationships, the power density in the cavity can be expressed as

$$S_c = \frac{\sigma_t \lambda Q}{2\pi V} S_i \tag{71}$$

Using the power conservation relationship, one can also write

$$P_w = \sigma_p S_i = \sigma_t \frac{\sigma_w}{\sigma_T} S_i \tag{72}$$

and thus

$$\sigma_p = \sigma_t \frac{\sigma_w}{\sigma_T} \ . \tag{73}$$

As pointed out by Lee and Yang [18], the factor σ_w / σ_T is the fraction of the total power, P_T, being absorbed by the wire. If, for simplicity, the only loss mechanisms are absorption by the wire and reradiation through the aperture, then $\sigma_T = \sigma_w + \sigma_r$ and

$$\sigma_p = \sigma_t \left[\frac{\sigma_w}{\sigma_w + \sigma_r} \right] . \tag{74}$$

Certain limiting situations become clear. Consider, for example, the case when the length of the wire is long compared with the aperture size and $\sigma_w > \sigma_r$. In this case $\sigma_p \simeq \sigma_t$ which indicates that essentially all of the energy being transmitted through the aperture is absorbed by the wire. In this case the Q of the cavity becomes

$$Q \simeq \frac{\omega W_w}{P_w} . \tag{75}$$

Both the Q-factor of the cavity and the power density, S_c, are much smaller in this case than the values that would be obtained by assuming that the cavity was lossless. This illustrates the hazard of using a two-step process (ie. firstly, to calculate fields inside a lossless structure and then, secondly, to use these fields to estimate coupling). It is clear that the two-step process will overestimate the coupling and violate the principle of energy conservation. If $\sigma_w < \sigma_r$, on the other hand, then the two-step process gives the correct result since the wire does not significantly load the cavity and reduce the internal field.

Lee and Yang [18] have used approximate expressions for the relevant cross-sections in (74) to determine a bound on the coupling to a wire inside a cavity with a narrow slot and have compared these results with experiment. As shown in Figure 9, this procedure provided good agreement in the cases reported by Lee and Yang.

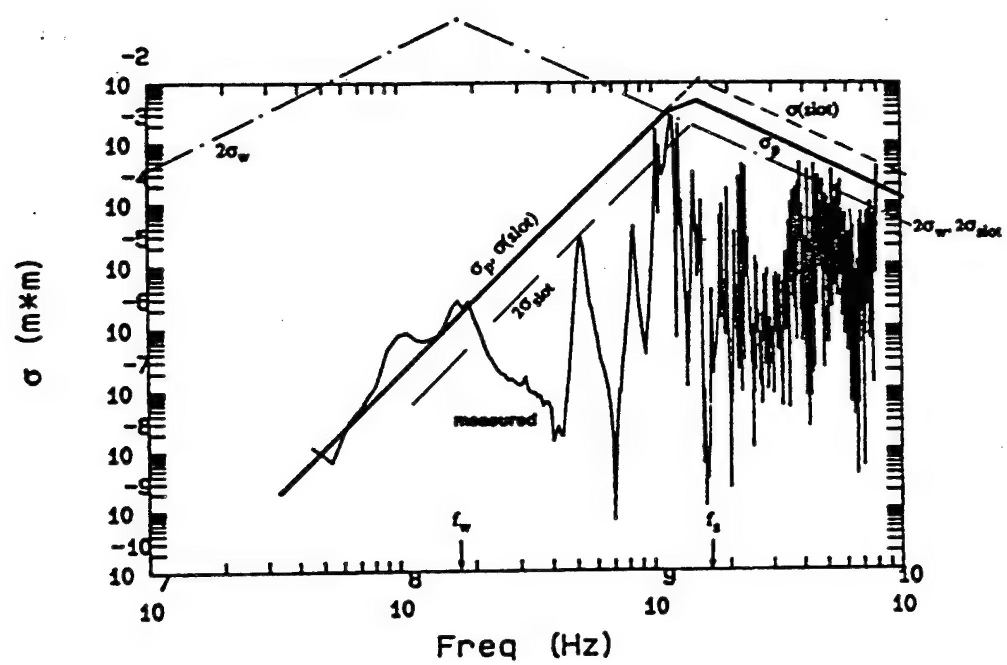


Figure 9 - Comparison of Measured Data [18] with Bounding Estimate

4. SUMMARY AND CONCLUSIONS

This report reviews the use of bounds to estimate:

a) the amount of energy that can be transmitted through an aperture;

b) the fields inside of a cavity with one or more apertures; and

c) the coupling of energy to wires inside of a cavity with apertures.

Because it is virtually impossible to model all of the detail of any real-world system, the approach examined in this report attempts to define the maximum energy or power that can couple to an electronic system in terms of an upper bound that is independent of most of the system details.

The first portion of this report reviews the methodology that has been developed by Lee [2] and Warne and Chen [3,4] to define a bound on the energy that can be transmitted through an aperture when the incident field takes the form of a step function or a double-exponential pulse. Such a bound is clearly of interest as, from the principle of energy conservation, this bound represents an upper limit to the energy that can couple to electronic systems in the interior of the structure containing the aperture. Ideally, however, one would hope to be able to define a tighter bound to prevent overhardening of the equipment if the information is used for this purpose. We have extended the work of Warne and Chen to define bounds for the energy that can be coupled from a damped-sine pulse, a rectangular RF pulse and an inverse-exponential pulse. These results have not been reported previously in the literature.

In the second part of the report, we show how a power conservation approach can be used to estimate the fields inside a lossless cavity with apertures. The subject is of interest as it offers an alternative to the computationally intensive numerical approach that is often used. Our work in this area is an extension of the ideas that have been presented by Lee and Yang [18] and Hill et al [20]. Using the power conservation approach the following conclusions have been reached:

a) at steady state, the shielding effectiveness offered by the cavity is, to first order, independent of aperture size and cavity Q; and

b) the minimum shielding effectiveness is generally of the order of -6 dB provided the frequency of the incident wave is above cavity resonance.

The final part of the report reviews the use of the power conservation approach to define a closer bound on the energy that can be coupled to a wire or some other absorber inside a cavity with an aperture. This is essentially a summary of the recent work of Lee and Yang [18]. Lee and Yang have shown that, by using approximate coupling cross-sections for interior absorbers as well as the apertures, it is possible to define a much closer bound than that based on the bound for energy transmitted through an aperture.

In conclusion, we have found that the use of energy bounds represents a useful approach for the estimation of interior fields and coupling to wires inside a structure with apertures. The approach has potential for simplifying the hardening of electronic systems.

5. REFERENCES

1. M. Burton and S. Kashyap, "Computer solution for field penetration into the bridge of the CPF", DREO Report 1075, 1991.
2. K.S.H. Lee, "Relations between electric and magnetic polarizabilities and other related quantities", Radio Science, 22, 1235 (1987).
3. L.K. Warne and K.C. Chen, "A bound on EMP coupling", IEEE Trans. on EMC, 32, 217 (1990).
4. L.K. Warne and K.C. Chen, "A bound on aperture coupling from realistic EMP", IEEE Trans. on EMC, 36, 149 (1994).
5. W.F. Crevier and J.R. Auton, "An improved limit for the energy radiated through an aperture by a finite rise time pulse", JAYCOR/SB 92-007, 1992. report prepared for DNA.
6. R.F. Harrington, Time Harmonic Electromagnetic Fields, McGraw-Hill Book Company, New York, 1961. p. 106.
7. J. van Bladel and C.M. Butler, "Aperture Problems", Proc. NATO Adv. Study Inst. on Theoretical Methods for Determining the Interaction of Electromagnetic Waves with Structures, J. Skwirzynski, Ed., Sythoff and Noordhoff International Publishers, 1979.
8. C.M. Butler, Y. Rahmat-Samii and R. Mittra, "Electromagnetic Penetration Through Apertures in Conducting Surfaces", IEEE Trans. on Antennas and Propagation, AP-26, 82 (1978).
9. J.D. Jackson, Classical Electrodynamics, John Wiley and Sons, New York, 1952.
10. J. van Bladel, Electromagnetic Fields, McGraw-Hill, New York, 1964.
11. R.E. Collin, Field Theory of Guided Waves, McGraw-Hill, New York, 1960, pp. 294-298.
12. F. De Meulenaere and J. van Bladel, "Polarizability of some small apertures", IEEE Trans. on Antennas and Propagation, 25, 198 (1977).
13. M. Born and E. Wolf, Principles of Optics, Pergamon, New York, 1980.
14. S. Haykin, Communication Systems, John Wiley and Sons, New York, 1978.
15. L.D. Landau and E.M. Lifshitz, Electrodynamics of Continuous Media, Pergamon, New York, 1960.
16. H.A. Bethe, "Theory of diffraction by small holes", Phys. Rev., 66, 163 (1944).
17. C.J. Bouwkamp, "Diffraction Theory", Rep. Prog. Phys., 17, 35 (1954).
18. K.S.H. Lee and F-C. Yang, "Trends and bounds in RF coupling to a wire inside a slotted cavity", IEEE Trans. on EMC, 34, 154 (1992).
19. K.S.H. Lee (Ed.), EMP Interaction: Principles, Techniques and Reference Data, Hemisphere Publishing Corp., Washington, 1986.

20. D.A. Hill, M.T. Ma, A.R. Ondrejka, B.F. Riddle, M.L. Crawford and R.T. Johnk, "Aperture excitation of electrically large, lossy cavities", IEEE Trans. on EMC, 36, 169 (1994).
21. J.A. Stratton, Electromagnetic Theory, McGraw-Hill, New York, 1941.
22. T.B.A. Senior and G.A. Desjardins, "Electromagnetic field penetration into a spherical cavity", IEEE Trans. on EMC, 16, 205 (1974).
23. H.A. Mendez, "On the theory of low-frequency excitation of cavity resonators", IEEE Trans. on MTT, 18, 444 (1970).
24. S. Kashyap, unpublished results.

APPENDIX A

In this appendix we evaluate the integral from Section 2.4.3:

$$I = \int_{-\infty}^{+\infty} \frac{S(\omega)}{(a^2-\omega^2+\omega_0^2)^2 + 4a^2\omega^2}\, d\omega$$

$$= \int_{-\infty}^{+\infty} \frac{S(\omega)}{[\omega-(\omega_0+ia)]^2\,[\omega-(\omega_0-ia)]^2}\, d\omega \; .$$

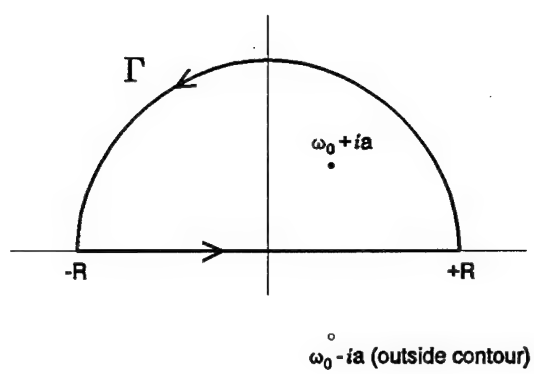
Consider the contour integral

$$\int_\Gamma f(z)\,dz \; ,$$

where

$$f(z) = \frac{S(\omega)}{[z-(\omega_0+ia)]^2\,[z-(\omega_0-ia)]^2} \quad ,$$

and the contour Γ is



The function f is then analytic inside the contour Γ except for the pole of order 2 at $\omega_0 + ia$. The path integral on the semicircle $\rightarrow 0$ as $R \rightarrow +\infty$. Thus by the residue theorem

$$
\begin{aligned}
\mathrm{I} &= \lim_{R \to +\infty} \int_{\Gamma} f(z)\,dz \\
&= 2\pi i \lim_{\omega \to \omega_0 + ia} \left\{ \frac{d}{d\omega} \frac{S(\omega)}{[\omega - (\omega_0 - ia)]^2} \right\} \\
&= 2\pi i \left\{ \frac{S'(\omega_0 + ia)}{(2ia)^2} - \frac{2S(\omega_0 + ia)}{(2ia)^3} \right\} \\
&= \frac{\pi}{2a^2} \left\{ \frac{S(\omega_0 + ia)}{a} - iS'(\omega_0 + ia) \right\} .
\end{aligned}
$$

Therefore

$$
\begin{aligned}
W_{tot} &= \frac{2\; E_0^2\; c^2\; \omega_0^2}{\eta_0} Re\{\mathrm{I}\} \\
&= \frac{\pi\; E_0^2\; c^2\; \omega_0^2}{\eta_0\; a^2} Re \left\{ \frac{S(\omega_0 + ia)}{a} - iS'(\omega_0 + ia) \right\} .
\end{aligned}
$$

APPENDIX B

In this appendix we evaluate the three integrals from Section 2.4.4:

$$I_1 = \int_{-\infty}^{+\infty} \frac{\sin^2[a(\omega-\omega_0)]}{(\omega-\omega_0)^2}\, S(\omega)\, d\omega \ ,$$

$$I_2 = \int_{-\infty}^{+\infty} \frac{\sin^2[a(\omega+\omega_0)]}{(\omega+\omega_0)^2}\, S(\omega)\, d\omega \ ,$$

and

$$I_3 = \int_{-\infty}^{+\infty} \frac{2\,\sin[a(\omega-\omega_0)]\,\sin[a(\omega+\omega_0)]}{\omega^2-\omega_0^2}\, S(\omega)\, d\omega \ .$$

B.1

Using the substitution $x = \omega - \omega_0$ and the identity

$$\sin^2(ax) = Re\left[\frac{1}{2}(1-e^{2iax})\right] ,$$

one obtains

$$I_1 = Re\int_{-\infty}^{+\infty} \frac{1-e^{2iax}}{2x^2}\, S(x+\omega_0)\, dx \ .$$

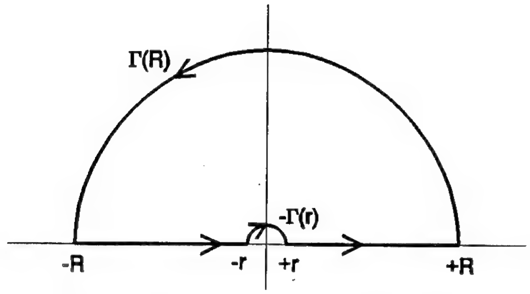
Consider the contour integral

$$\int_{\Gamma} f(z)dz \ ,$$

where

$$f(z) = \frac{(1-e^{2iaz})S(z+\omega_0)}{2z^2} \ ,$$

and the contour Γ is



Γ = [-R,-r] $\cup$ -Γ(r) $\cup$ [r,R] $\cup$ Γ(R)

The limit

$$\lim_{z \to 0} z\, f(z) = \lim_{z \to 0} \frac{1 - e^{2iaz}}{2z} S(z + \omega_0)$$

$$= -iaS(\omega_0) \ ,$$

implies that

$$\lim_{r \to 0} \int_{-\Gamma(r)} f(z)dz = (i)[-iaS(\omega_0)](-\pi)$$

$$= -a\pi S(\omega_0) \ .$$

Similar to Appendix A,

$$\lim_{R \to +\infty} \int_{\Gamma(R)} f(z)dz = 0 \ .$$

The contour Γ encloses no singularities of f ; so by Cauchy's theorem

$$\int_{\Gamma} f(z)dz = 0 \ .$$

Since

$$\int_{\Gamma} = \int_{-R}^{-r} + \int_{-\Gamma(r)} + \int_{r}^{R} + \int_{\Gamma(R)} \ ,$$

taking $r \to 0$ and $R \to +\infty$ one obtains

$$\int_{-\infty}^{+\infty} f(x)\,dx - a\pi S(\omega_0) = 0 .$$

Therefore

$$I_1 = Re\left[\int_{-\infty}^{+\infty} f(x)\,dx\right] = Re\;[a\pi S(\omega_0)] .$$

B.2

Similarly, the integral I_2, with the substitution $x = \omega + \omega_0$, etc., can be shown to have the value

$$I_2 = Re\int_{-\infty}^{+\infty} \frac{1-e^{2iax}}{2x^2}\, S(x-\omega_0)\; dx = Re\;[a\pi S(-\omega_0)] .$$

B.3

Using the identity

$$2\sin[a(\omega-\omega_0)]\sin[a(\omega+\omega_0)] = Re\left[e^{2ia\omega_0} - e^{2ia\omega}\right],$$

one obtains

$$I_3 = Re\left\{\int_{-\infty}^{+\infty} \frac{e^{2ia\omega_0} - e^{2ia\omega}}{\omega^2-\omega_0^2}\, S(\omega)\; d\omega\right\} .$$

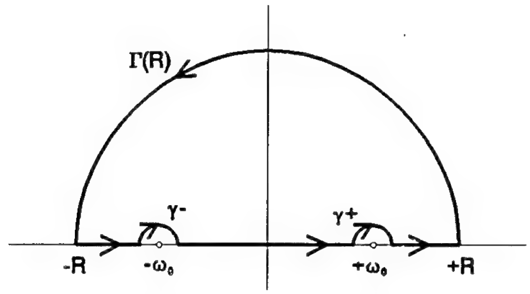
Consider the contour integral

$$\int_\Gamma f(z)\,dz ,$$

where

$$f(z) = \frac{e^{2ia\omega_0} - e^{2iaz}}{z^2-\omega_0^2}\, S(z) ,$$

and the contour Γ is



$$\Gamma = [-R, -\omega_0 - r] \cup \gamma- \cup [-\omega_0 + r, \omega_0 - r] \cup \gamma+ \cup [\omega_0 + r, R] \cup \Gamma(R)$$

Similar to the calculation in B.1, the limit

$$\lim_{z \to -\omega_0} (z + \omega_0)\, f(z) = \lim_{z \to -\omega_0} \frac{e^{2ia\omega_0} - e^{2iaz}}{z - \omega_0}\, S(z)$$

$$= \frac{2i \sin[2a\omega_0]}{-2\omega_0}\, S(-\omega_0)\ ,$$

implies that

$$\lim_{r \to 0} \int_{\gamma-} f(z)\, dz = (i) \frac{2i \sin[2a\omega_0]}{-2\omega_0} S(-\omega_0)(-\pi)$$

$$= -\frac{\pi \sin[2a\omega_0]}{\omega_0}\, S(-\omega_0)\ .$$

On the other small semicircle, the limit

$$\lim_{z \to +\omega_0} (z - \omega_0)\, f(z) = \lim_{z \to +\omega_0} \frac{e^{2ia\omega_0} - e^{2iaz}}{z + \omega_0}\, S(z) = 0\ ,$$

implies that

$$\lim_{r \to 0} \int_{\gamma+} f(z)\, dz = 0\ .$$

Also,

$$\lim_{R \to +\infty} \int_{\Gamma(R)} f(z)\, dz = 0\ .$$

The contour Γ encloses no singularities of f ; so by Cauchy's theorem

$$\int_{\Gamma} f(z)dz = 0 .$$

Since

$$\int_{\Gamma} = \int_{-R}^{-\omega_0 - r} + \int_{\gamma-} + \int_{-\omega_0 + r}^{\omega_0 - r} + \int_{\gamma+} + \int_{-\omega_0 + r}^{R} + \int_{\Gamma(R)} ,$$

taking $r \to 0$ and $R \to +\infty$ one obtains

$$\int_{-\infty}^{+\infty} f(x)dx - \frac{\pi \sin[2a\omega_0]}{\omega_0} S(-\omega_0) = 0 .$$

Therefore

$$I_3 = Re\left[\int_{-\infty}^{+\infty} f(x)dx\right] = Re\left[\frac{\pi \sin[2a\omega_0]}{\omega_0} S(-\omega_0)\right].$$

Finally

$$W_{tot} = \frac{2 E_0^2 c^2}{\eta_0} Re\{I_1 + I_2 + I_3\}$$

$$= \frac{2 \pi c^2 E_0^2}{\eta_0} Re \left\{ aS(\omega_0) + \left[a + \frac{\sin[2a\omega_0]}{\omega_0}\right] S(-\omega_0)\right\} .$$

APPENDIX C

In this appendix we evaluate W_{tot} from Section 2.4.5:

$$W_{tot} = \frac{4\, c^2\, E_0^2\, \pi^2\, (\beta-\alpha)^{\frac{2\alpha}{\beta}-2}\, \alpha^{-\frac{2\alpha}{\beta}}}{\eta_0} \int_{-\infty}^{+\infty} \frac{Re[S(\omega)]}{\cosh\left(\frac{2\,\omega\,\pi}{\beta}\right) - \cos\left(\frac{2\,\alpha\,\pi}{\beta}\right)}\, d\omega \ .$$

A few substitutions will simplify the integral.

Let $t = \frac{2\,\alpha\,\pi}{\beta} - \pi$, then $\cos\left(\frac{2\,\alpha\,\pi}{\beta}\right) = -\cos\left(\frac{2\,\alpha\,\pi}{\beta} - \pi\right) = -\cos t$. Note that

$$t = \left(\frac{2 \times 10.3 \times 10^8}{10.34 \times 10^8} - 1\right)\pi \approx 0.992\,\pi \ .$$

Further, let $x = \frac{2\,\omega\,\pi}{\beta}$ and $\hat{S}(x) = Re[S(\omega)] = Re[S(\frac{\beta}{2\pi}x)]$. Since the integrand is an even function of x ,

$$\int_{-\infty}^{+\infty} = 2\int_{0}^{+\infty} ,$$

and the Maclaurin series of $\hat{S}$ has only even powers of x :

$$\hat{S}(x) = \sum_{k=0}^{+\infty} \frac{\hat{S}^{(2k)}(0)}{(2k)!}\, x^{2k} \ .$$

With these substitutions

$$W_{tot} = \frac{4\, c^2\, E_0^2\, \pi\, (\beta-\alpha)^{\frac{2\alpha}{\beta}-2}\, \alpha^{-\frac{2\alpha}{\beta}}\, \beta}{\eta_0} \int_{0}^{+\infty} \frac{\hat{S}(x)}{\cosh x + \cos t}\, dx \ .$$

The problem is then reduced to the evaluation of integrals of the form

$$I_k = \int_0^{+\infty} \frac{x^{2k}}{\cosh x + \cos t}\, dx\,, \qquad k = 0,1,2,...$$

Let us use the integral

$$I_0 = \int_0^{+\infty} \frac{dx}{\cosh x + \cos t}$$

as an illustration. Now

$$I_0 = 2\int_0^{+\infty} \frac{dx}{e^x + e^{-x} + 2\cos t} = \int_{-\infty}^{+\infty} \frac{e^x\, dx}{e^{2x} + 2\cos t\, e^x + 1}\,.$$

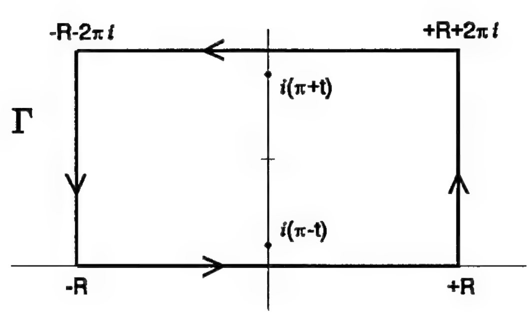
Consider the contour integral

$$\int_\Gamma f(z)dz\,,$$

where

$$f(z) = \frac{e^{az}}{e^{2z} + 2\cos t\, e^z + 1}\,,$$

and the contour Γ is



The function f has two simple poles inside the contour Γ at $i(\pi - t)$ and $i(\pi + t)$. This is because

$$\begin{aligned} & e^{2z} + 2\cos t\, e^{z} + 1 = 0 \ ; \\ \Rightarrow \qquad & e^{z} = -\cos t \pm i\sin t \\ & \quad = \cos(\pi \mp t) + i\sin(\pi \mp t) \ ; \\ \Rightarrow \qquad & z = i(\pi - t) \ , \ i(\pi + t) \ . \end{aligned}$$

Note that

$$f(x + i2\pi) = e^{2\pi i a} f(x) \ ,$$

and the integrals on the vertical paths $\to 0$ as $R \to +\infty$. Thus

$$\begin{aligned} \lim_{R\to+\infty} \int_{\Gamma} f(z)\,dz &= \lim_{R\to+\infty} \left\{ \int_{-R}^{+R} f(x)\,dx + \int_{+R}^{-R} e^{2\pi i a} f(x)\,dx \right\} \\ &= \left(1 - e^{2\pi i a}\right) \int_{-\infty}^{+\infty} f(x)\,dx \ . \end{aligned}$$

The residues are

$$\begin{aligned} Res\left[f\,; i(\pi - t)\right] &= \lim_{z \to i(\pi - t)} \frac{[z - i(\pi - t)]\, e^{z(a-1)}}{e^{z} + e^{-z} + 2\cos t} \\ &= \frac{e^{i(\pi - t)(a-1)}}{i2\sin t} \ ; \end{aligned}$$

and

$$Res\left[f\,; i(\pi + t)\right] = \frac{-e^{i(\pi + t)(a-1)}}{i2\sin t} \ .$$

Therefore by the residue theorem

$$\begin{aligned} \int_{-\infty}^{+\infty} f(x)\,dx &= \frac{2\pi i}{1 - e^{2\pi i a}} \ \frac{1}{i2\sin t} \left[e^{i(\pi - t)(a-1)} - e^{i(\pi + t)(a-1)} \right] \\ &= \frac{\pi}{\sin t} \ \frac{-1}{\sin(a\pi)} \left[e^{i\pi(2a-1)} \sin\!\left(t(1-a)\right) \right] \ . \end{aligned}$$

Finally,

$$I_0 = \lim_{a\to 1} \int_{-\infty}^{+\infty} f(x)\,dx = \frac{-\pi}{\sin t}\left[\frac{-\cos t\,(1-a)\,(-t)}{\pi\,\cos(a\pi)}\right] = \frac{t}{\sin t} \ .$$

Using a similar technique, and taking *extreme* care of the algebra, one obtains

$$I_1 = \int_0^{+\infty} \frac{x^2\,dx}{\cosh x + \cos t} = \frac{t}{\sin t}\,\frac{\pi^2 - t^2}{3} \ ,$$

$$I_2 = \int_0^{+\infty} \frac{x^4\,dx}{\cosh x + \cos t} = \frac{t}{\sin t}\left[\frac{\pi^2 - t^2}{3}\right]\left[\frac{7\pi^2 - 3t^2}{5}\right] \ ,$$

$$\vdots$$

Taking the first three terms of the Maclaurin series of $\hat{S}$, one then has

$$W_{tot} \approx \frac{4\,c^2\,E_0^2\,\pi^2\,(\beta-\alpha)^{\frac{2\alpha}{\beta}-2}\,\alpha^{-\frac{2\alpha}{\beta}}\,(2\alpha-\beta)}{\eta_0\,\sin\left((\frac{2\alpha}{\beta}-1)\pi\right)} \times$$

$$\left\{\hat{S}(0) + \frac{\hat{S}^{(2)}(0)}{2}\,\frac{\pi^2 - t^2}{3} + \frac{\hat{S}^{(4)}(0)}{4!}\left[\frac{\pi^2 - t^2}{3}\right]\left[\frac{7\pi^2 - 3t^2}{5}\right] + \ldots\right\} \ .$$

Since $t \approx 0.992\,\pi$, the high-order terms are relatively small. Thus taking only the first term,

$$W_{tot} \approx \frac{4\,c^2\,E_0^2\,\pi^2\,(\beta-\alpha)^{\frac{2\alpha}{\beta}-2}\,\alpha^{-\frac{2\alpha}{\beta}}\,(2\alpha-\beta)}{\eta_0\,\sin\left((\frac{2\alpha}{\beta}-1)\pi\right)}\,Re[S(0)] \ .$$

DOCUMENT CONTROL DATA

(Security classification of title, body of abstract and indexing annotation must be entered when overall document is classified)

1. ORIGINATOR (the name and address of the organization preparing the document. Organizations for whom the document was prepared, (e.g. Establishment sponsoring a contractor's report, or tasking agency are entered in section 8.) Defence Research Establishment Ottawa 3701 Carling Avenue Ottawa, Ontario, Canada K1A 0Z4	2. SECURITY CLASSIFICATION (overall security classification of the document including special warning terms if applicable) UNCLASSIFIED

3. TITLE (the complete document title as indicated on the title page. Its classification should be indicated by the appropriate abbreviation (S, C or U in parentheses after the title.)

Use of Energy Bounds to Estimate Fields and Coupling Inside a Cavity with Apertures (U)

4. AUTHORS (Last name, first name, middle initial)

Gardner, C.L., and Louie, A.

5. DATE OF PUBLICATION (month and year of publication of document) MAR/95	6a. NO. OF PAGES (total containing information. Include Annexes, Appendicies, etc.) 41	6b. NO. OF REFS (total cited in document) 24

7. DESCRIPTIVE NOTES (the category of the document, e.g. technical report, technical note or memorandum. If appropriate, enter the type of report, e.g. interim, progress, summary, annual or final. Give the inclusive dates when a specific reporting period is covered.)

DREO Report 1261

8. SPONSORING ACTIVITY (the name of the department project office or laboratory sponsoring the research and development. Include the address.)

Defence Research Establishment Ottawa
3701 Carling Avenue
Ottawa, Ontario, Canada K1A 0Z4

9a. PROJECT OR GRANT NO. (if appropriate, the applicable research and development project or grant number under which the document was written. Please specify whether project or grant) 041LT	9b. CONTRACT NO. (if appropriate, the applicable number under which the document was written)
10a. ORIGINATOR'S DOCUMENT NUMBER (the official document number by which the document is identified by the originating activity. This number must be unique to this document.) DRP 95-301	10b. OTHER DOCUMENT NOS. (Any other numbers which may be assigned this document either by the originator or by the sponsor) DREO REPORT 1261

11. DOCUMENT AVAILABILITY (any limitations on futher dissmination of the document, other than those imposed by security classification)

(X) Unlimited distribution
( ) Distribution limited to defence departments and defence contractors; futher distribution only as approved
( ) Distribution limited to defence departments and Canadian defence contractors; futher distribution only as approved
( ) Distribution limited to goverment departments and agencies; futher distribution only as approved
( ) Distribution limited to defence departments; Futher distribution only as approved
( ) Other (please specify):

12. DOCUMENT ANOUNCEMENT (any limitation to the bibliographic announcement of this document. This will normally correspond to the Document Availability (11). However, where further distribution (beyond the audience specified in 11) is possible, a wider announcement audience may be selected.)

13. ABSTRACT (a brief and factual summary of the documents. It may also appear elsewhere in the body of the document itself. It is highly desirable that the abstract of classified documents be unclassified. Each paragraph of the abstract shall begin with an indication of the security classification of the information in the paragraph (unless the document itself is unclassified) represented as (S), (C), or (U). It is not necessary to include here abstracts in both official languages unless the text is bilingual.

(U) This report investigates the use of energy bounds to estimate: the amount of energy that can be transmitted through an aperture when exposed to an EM pulse; the fields inside a cavity with one or more apertures; and the coupling of energy that can be coupled from a damped -sine wave, a rectangular RF pulse and an inverse-exponential pulse.

(U) It is shown that the use of energy bounds is a useful approach to the estimation of interior fields and coupling to systems inside a structure with apertures. This approach has potential for simplifying the hardening of electronic systems.

14. KEYWORDS, DESCRIPTORS or IDENTIFIERS (technically meaningful terms or short phrases that characterize a document and could be helpful in cataloguing the document. They should be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location may also be included. If possible keywords should be selected from a published thsaurus. e.g. Thesaurus of Engineering and Scientific Terms (TEST) and that thesaurus -identified. If it is not possible to select indexing terms which are Unclassified, the classification of each should be indicated as with the title.)

energy bounds
aperture
electromagnetic pulse
rectangular Rf pulse
coupling